

Concept of momentum-less bodies and a suggestion for its experimental verification using ultra-cold atoms

Sanjay M Wagh*

Central India Research Institute, Post Box 606,

Laxminagar, Nagpur 440 022, India

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General Principle of Relativity unequivocally supports the notion of momentum-less energy for bodies (energy-quanta) moving at the *same* or *constant* speed relative to all the reference systems. In this communication, we point out that whether energy-quantum is a momentum-less body or not is verifiable using ultra-cold atoms trapped in an optical lattice, perhaps with some minor modifications to the existing such experimental setups.

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INTRODUCTION

If the speed of a certain body is the same, or a constant, in relation to all the reference systems, then the inertia of such a body must vanish, and so must its momentum, as a product of its inertia and speed, in all reference systems. This is, clearly, within the overall framework of Einstein's General Principle of Relativity.

If bodies exist with speed being constant relative to all the reference systems, then we have two types of bodies: inertia-less bodies and bodies with inertia or material bodies.

Thence, consider that inertia-less body merges with or emerges from a material body. Material body will then change in characteristics (energy etc.) by their values non-vanishing for the inertia-less body, while those characteristics (momentum etc.) vanishing for inertia-less body will not have changed for it. In other words, no momentum is imparted to a material body in its interaction with a inertia-less body.

To the best of the knowledge of the author, no attention whatsoever appears to have been paid to this aspect of the General Principle of Relativity. No references to any discussions of this situation of momentum-less bodies and its consequences are known [6] to the author.

We are thus led to the existence of the inertia-less and, therefore, momentum-less, bodies within the overall premise of the concepts associated with the General Principle of Relativity.

For implementing this relativity principle, one usually focuses only on the "relative" motions of bodies or on binary relations between bodies. This is insufficient basis: Newtonian framework also has quantifiers of motion (distance, speed etc.) as binary relations of bodies, but its laws do not conform with this principle.

Therefore, we adopt [1] two principles: first is the *General Relativity Principle* that there are none preferred motions of any reference bodies for formulating the Laws of Nature, and second is the *Universality of Relativity Principle* that there is none preferred mathematical representation of bodies for formulating such Laws.

This is the Universal Relativity [1], whose framework capable of dealing with all the mathematical structures, is of the Category Theory [2].

Although we will not discuss the mathematical details of the Universal Relativity, we point out here that a "momentum-less body" is one of its experimentally testable predictions.

Conceptually, a material body does not, relative to an observer, "move" unless its energy is "larger" than its inertia (*ie*, its rest energy). Then, by the merger with a momentum-less body, energy of a material body, at rest in relation to the observer, "grows" larger than its inertia. With it, a material body moves. Only the bodies with non-vanishing inertia may collide, while in motion, to exchange momentum between them.

This is "testable" for the microscopic bodies like an atom. On the absorption of a momentum-less energy-quantum by an atom, we should "observe" that atomic momentum changes in the direction only of its existing motion. Such an experimental test of the momentum-less bodies seems within the capabilities of present experiments.

CONSEQUENCES OF THE NOTION OF MOMENTUM-LESS ENERGY-QUANTUM

In the general context, we note that "statistical" considerations do not distinguish momentum-less radiation from the usual notion of radiation with momentum $\vec{p}_\gamma = (h\nu/c)\hat{c}$ [3], symbols having usual meanings. This is seen as follows.

In the setting of the Category Theory, consider an abelian group of order n as the only object G_n of its category \mathbf{G}_n with the group elements being the categorical arrows from G_n to G_n . This is an additive category, with the group identity being its zero arrow or additive identity [1, 2].

Any measure functor [7] [1] from category \mathbf{G}_n to additive category \mathbb{R}^+ of the group (or the monoid) of addition of real numbers maps zero arrow to the additive identity $0 \in \mathbb{R}^+$. Other arrows of the category \mathbf{G}_n can then be mapped to $n - 1$ distinct possible real numbers. These are then “discrete” values of, what we call as, the *characteristics of individuality* of object G_n .

Measure functors to different additive categories (including even \mathbf{G}_n) can provide (conceptually) distinct characteristics of individuality of object, like the group object G_n .

Characteristics of individuality are “intrinsic” properties of the object, like its mass, energy, electric charge, spin, \dots Because of their functorial nature, there exist relations between these values. For example, energy can be “decomposed” into (intrinsic) parts such as rest, spin, \dots and, when allowed, (an extrinsic) kinetic part [8].

Now, we can consider an object with zero inertia, vanishing momentum, non-vanishing energy, spin one \dots [9]. We call it as a *momentum-less energy-quantum*, and call a particle with momentum $h\nu/c$ and spin one as a *photon*.

When we treat SG_nS^{-1} as another object G'_n , with S being an invertible matrix, we form a new category \mathbb{G}_n , with this similarity transformation being an arrow from G_n to G'_n .

We can define measures over this new category as well. Then, a similarity transformation arrow of G_n to G'_n has measure of distance separating the objects. Any change in the distance measure is [1] a similar functor.

Characteristics of individuality of G_n , as were its in the category \mathbf{G}_n , exist also within the new-made category \mathbb{G}_n as the group G_n is [1] “entirely included” within it.

There also is a functor defining [1] “time” within the context of this category \mathbb{G}_n . It is a universal label of all the sequences of its objects.

“Time” is then absolute: changes to objects of the category do not cause changes to the time label. However, time is defined on the basis of categorical objects and, thence, has no existence or meaning independently of the categorical objects. Speed, momentum, \dots are notions available for the objects of this category \mathbb{G}_n , though vanishing [10] for the only object of \mathbf{G}_n .

Category \mathbb{G}_n is a “gas” of indistinguishable objects possessing, of course, their individuality. We may include energy-quanta within this category, if desired. This is a general feature not restricted only to the category \mathbb{G}_n . We now use it to treat categorical objects statistically.

For this purpose, we may imagine that we are considering objects of real-valued characteristics (some intrinsic), and are bookkeeping (additive) changes to them when these objects interact with momentum-less bodies. These characteristics are measures and, hence, additive.

Thus, let the total energy of an object be the sum $E^T = E^{KE} + E^0 + E^i$, where E^{KE} is the kinetic energy equaling $\frac{1}{2}mv^2$, E^0 is the rest energy equaling mc^2 , and E^i is “intrinsic” characteristic energy of that object.

For momentum-less energy-quantum, we have $E^0 = 0$, $E^{KE} = 0$, $E^i = \epsilon$ in appropriate units. Its energy is therefore entirely of “intrinsic” type. As another example, an electron has $E^0 = m_e c^2$, $E^{KE} = \frac{1}{2}m_e v^2$, $E^i = \frac{1}{2}\epsilon$ in appropriate units. Any material body then has an “extrinsic” component of energy, the kinetic energy.

Clearly, the energy absorbed or radiated as an energy-quantum by an atom with $E \geq E^0$ changes its only extrinsic energy - the kinetic energy, and may change those intrinsic contributions for which energy-quantum has non-zero values. All the other contributions do not change.

Using units of energy which are the same for all these components, we then get, in general:

$$E^T = E^{KE} + n\epsilon$$

where ϵ is the common unit, and n is allowed only integer or half-integer values.

Considering then a gas of identical particles and distributing them canonically over the N number of boxes of kinetic energy with the total kinetic energy of the particles being E , the average kinetic energy, ie, E/N , is, with total energy in the j -th box being written $E_j^T = j\epsilon$, given by

$$\bar{E} = E/N = (1 - n)\epsilon + \frac{\epsilon}{e^{\epsilon/kT} - 1} \quad (1)$$

with all *a priori* probabilities being equal.

This result is not surprising, for it will obtain when the objects are statistically independent, and when the atomic energy has a non-zero minimum value. Clearly, this is the same result as that for a photon with momentum $p = h\nu/c$, and $n = 1$, providing us Planck’s results.

We now consider momentum fluctuations of an atom at equilibrium in a bath of momentum-less energy-quanta.

First, we emphasize that there exists a definite “direction of motion” for a momentum-less energy-quantum, just as it exists for any other categorical object. Therefore, the emission and absorption of an energy-quantum are *directional* processes for the energy-quantum.

Even in the categorical description, the distance, as a binary relation of objects, vanishes only for a pair of identical objects. Thence, in an absorption process, an energy-quantum and an object absorbing it, both, must become the same categorical object, albeit with the characteristics of individuality obtained from the addition of those of the “merging” objects.

Thence, consider now an atom, of non-vanishing mass m , inside a cavity containing momentum-less energy-quanta such that energy-quanta move and interact with the atom entirely independently of each other.

Let this be a categorical object having energy E' that is the sum of energies E and E_γ . The energy E_γ can now be carried by an energy-quantum that may move in any direction whatsoever, with the energy E being associated with another object. This is emission process.

Similarly, in an absorption process, an energy-quantum of energy E_γ coming from any direction can merge with an object of energy E to leave an object of energy $E' = E + E_\gamma$.

Categorically, all the directions are permissible for the motion of an energy-quantum in either process, and are equally likely on the basis of categorical considerations, no preferred direction exists for the motion of energy-quantum within our statistical considerations.

Emission and absorption of energy-quantum are *directional processes* for the atom as the change in its energy is only kinetic. Atomic momentum thus changes only along the direction of the existing motion of the atom.

Consider then a bath of energy quanta and an atom of mass m in it having velocity v_1 . As atom absorbs an energy-quantum of energy ϵ , let its energy $E_1 = \frac{1}{2}mv_1^2 + E^o + E^i$ change to $E_2 = \frac{1}{2}mv_2^2 + E^o + E^i$. Change in energy is related to that in momentum by $\delta P = \left(\frac{2}{v_1+v_2}\right)\delta E$.

Now, velocities v_1 and v_2 , hence, $v_1 + v_2$, can be reached not only by absorbing just a single energy-quantum of energy ϵ , but also in many different ways by absorbing *arbitrary* number of quanta of energies $\epsilon', \epsilon'', \dots$. Therefore, the quantity in the brackets in the above expression for δP must be a universal quantity for this process.

There is only one velocity which is universal, the speed of light c . Therefore, we set $\left(\frac{2}{v_1+v_2}\right) \equiv 1/c$ and, thereby, obtain $\delta P = \delta E/c$.

[This above is easily understood. Kinetic energy is a measure definable from the distance measure. Similarly, (the amplitude of) the momentum too is a measure definable from the distance measure. Any two such “functors” can be related only by a quantity that cannot be category specific or categorical object dependent. It must be universal relationship for the functors.]

We now distribute atom(s) in boxes of momenta under equilibrium with the bath of energy-quanta. Proceeding as before, and consistently with (1), we then obtain the average momentum of the atom:

$$\bar{P} = (1 - n)\frac{\epsilon}{c} + \frac{\epsilon/c}{e^{\epsilon/kT} - 1} \quad (2)$$

Again, it easily follows also that considerations of fluctuations to the momentum of the atom in an energy-quantum bath do not change from those for the “photons” even if the energy-quantum is momentum-less!

Therefore, momentum-less energy-quantum is a notion consistent with all the “usual” phenomena for Light or Radiation. Momentum of an energy-quantum is an “illusion” arising out of these facts then [11]. The momentum-less energy-quantum can, certainly erroneously, sometimes be viewed as a body with momentum $h\nu/c$.

Now, about phenomena that “can distinguish” momentum-less energy-quantum from our “usual” notion of a photon of momentum $h\nu/c$.

DISCUSSION

In a rarified collection of atoms that are ultra-cold, we may now be able to test the notion of momentum-less radiation by monitoring motions of the atoms of such a collection.

Let us imagine that atoms are “free” to move. Using a technique involving a CCD, let us “photograph” these atoms in sufficiently rapid successions before the atoms collide. In the process of CCD-imaging the atoms, we are imparting momenta to them using momentum-less energy-quanta, say, of energy in the optical range.

If this is correct, then atomic momenta change only in the directions of the already existing atomic motions. Comparison of CCD images should then reveal whether atoms, before they collide with one another, maintain their directions of motion across the CCD images or not.

Then, consider atoms, such as ^{88}Sr or neutral Caesium, super-cooled [4, 5] in Laser Traps. If such atoms are irradiated with short bursts of optical radiation, then verifying momentum-less nature of energy-quantum seems possible.

The author is not knowledgeable enough about the experimental setups to provide any further considerations than are given above, but notes that CCD data are obtained in them.

If the images of the atomic cloud are obtained in rapid successions, before its atoms collide, then comparisons of image data may “verify” if the atomic momenta change only in the directions of their existing motions on absorbing or emitting energy-quanta ...

In conclusion, we may express confidence that the notion of momentum-less energy-quantum will be the appropriate one. Certainly, our confidence emanates in the facts that this concept arises from the General Relativity Principle, and it is, by the Universality of Relativity Principle, independent of the mathematical representation of the physical bodies, which energy-quanta are.

Dedication:

This article and work are dedicated to Dr Tarak Kate, Prof A V Tankhiwale, Prof S S Srihande, and to many fond memories of (Late) Prof N V (alias Dada) Karbelkar, (Late) Prof V M Tak, and (Late) Prof A B Buche.

* Electronic address: cirinag'ngp@sancharnet.in

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- [6] Einstein could have associated momentum $h\nu/c$ with his light-quantum, when he associated energy $E = h\nu$ with it in 1905. Explicitly, Einstein did not do so for years [3, p. 408]!
Perhaps, General Principle of Relativity's natural implication of the momentum-less object held him back, at the subconscious levels, from associating momentum to light-quantum. Momentum $h\nu/c$ was explicitly associated with photon by Johannes Stark, firstly, in 1909 [3, p. 409].
In 1916, Einstein studied momentum fluctuations of an atom immersed in a radiation bath. It is only then he stated, still with a very careful selection of words, that “if a bundle of radiation causes a molecule to emit or absorb an energy amount $h\nu$, then a momentum $h\nu/c$ is transferred to the molecule, directed along the bundle for absorption and opposite the bundle for [induced] emission.”
- [7] A functor is a partial binary algebra preserving “ordinary function” from the collection of all the arrows of one to that of another category such that identity arrows of one are mapped to identity arrows of another category, and the composition of arrows of one category is mapped to the composition of mapped arrows. Additivity of a functor here is that of the additivity of families of categorical objects.
- [8] Note that there is no concept of potential energy in Universal Relativity, because each element of the “decomposition” of energy must also be a functorial measure.
- [9] There exist objects of vanishing inertia, vanishing momentum, non-vanishing energy and half-integer spin. Such objects are, however, outside of the scope of the present considerations.
- [10] Motion of one body is relative to another, and we have only one object in the category \mathbf{G}_n .
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